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Dynamic determination of DC motor parameters – simulation and testing

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***Abstract* – DC motors are widespread used in modern day applications of different kind, especially those that require a closed loop control. In order to use a DC motor in closed loop control systems, it is required to know the dynamic both electrical and mechanical parameters. Parameter estimation for DC motors, both brushed and brushless, is of equal importance in the field of motor control. Accurate determination of DC motor parameters is vital for a good control. The estimation of parameters could be done using several methods: using hybrid equivalent circuit model, step response, dynamic load variation, closed loop error approach, least square parametric estimation, least-squares approximation technique and a closed-loop disturbance observer. In the present paper, it is studied the dynamic mechanical parameter identification as a response to a step input signal and identified with a 2nd order dynamic system.**

Keywords- DC motor, transfer function, speed overshoot, inertial torque, visous friction coefficient

I. INTRODUCTION

DC motors are widespread used in modern day applications of different kind, especially those that require a closed loop control. Thus DC motors are encountered in: renewable energy production systems, vehicles with different purposes, medical equipments, robots, etc. Because of such wide use and the fact that DC motors are quite easy to control, in the research community, there is high interest for studying them.

In order to use a DC motor in closed loop control systems, it is required to know the dynamic both electrical and mechanical parameters. There is great interest in the identification procedures of electric motor parameters.

Parameter estimation for DC motors, both brushed and brushless, is of equal importance in the field of motor control. Accurate determination of DC motor parameters is vital for a good control [3][4][5][6]. The estimation of parameters could be done using several methods: using hybrid equivalent circuit model [7], step response [8], dynamic load variation [1], closed loop error approach [2], least square parametric estimation [5], least-squares approximation technique and a closed-loop disturbance observer [6].

In [2] the authors propose a Closed Loop Input Error (CLIE) algorithm. In their model there are two identical PD controllers. The inputs error between the systems that include the controllers are input signals for an identification algorithm that subsequently update the model parameters.

In [4] it was analysed the model and control of the pancake DC and it has been derived its nonlinear physical model. As the motor parameters were expected to be uncertain under different operating conditions, a robust second order sliding mode control scheme has been developed.

In [5] the authors presents an identification of the DC motor using the MATLAB *pem* function.

In [8], the author studies the parameter determination of a DC motor by factorization of the transfer function using LabView environment for simulation and calculation.

In the present paper, it is studied a parameter identification method as a response to a step input signal. The response parameters were measured, acquired and identified with the transfer function of a 2nd order dynamic system. Knowing the electrical parameters, from direct measurement, it could be determined the inertia torque and viscous friction coefficient. These parameters are very important in setting the speed and current regulators' parameters in a system with a DC motor [3].

Determining with good precision of DC motor parameters also has a good impact in the diagnosis of both the motor [9] and the whole system that includes it: the power converter [10] and the control structure.

The paper is structured in six chapters, covering the DC motor dynamic equations, equivalent schematic, parameter determination, experimental verification and conclusions.

II. DC MOTOR EQUATION

The schematic of a DC motor is indicated in Figure 1.

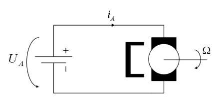

Figure 1. General schematic of permanent magnet DC motor

Using the notations in figure 1, it can be written the dynamic regime equations that describe the behaviour of a DC motor:

$$\begin{cases} u_A(t) = R_A i_A(t) + L_A \dfrac{di_A(t)}{dt} + e(t) \\ J\dfrac{d\Omega(t)}{dt} = m - m_f - m_s \end{cases} \tag{1}$$

Where:

U_A – voltage applied to motor's terminals
I_A – armature current
e – back emf
R_A, L_A – electric parameters of the motor: armature resistance and inductance
J, f – mechanical dynamic parameters: J – total inertial torque; f – viscous friction coefficient
m – electromagnetic torque
$m_f = f\cdot\Omega$ – viscous friction torque
m_s – kinetic friction torque

In equation (1) both the back emf and the electromagnetic torque depend direct in proportionality with speed and current:

$$\begin{cases} e = K\Phi\cdot\Omega \\ m = K\Phi\cdot i_A \end{cases} \tag{2}$$

In equation (2) $K\Phi$ is a proportional term, which in case of DC permanent magnet or more generally in shunt or independent configurations, is constant. Its value can be determined from the nominal values of the motor indicated in its nameplate:

$$K\Phi_n = \frac{U_{An} - R_A I_{An}}{\Omega_n} = \frac{U_{An} - R_A I_{An}}{\dfrac{2\pi\cdot n_n}{60}} \tag{3}$$

In equation (3) it is indicated the relation between the speed expressed in rad/s and rot/min. All values of equation (3) are indicated by the motor producer, and are known before any use of it.

In equation (1), by applying the Laplace transform it can be obtained the dynamic system equations of the motor:

$$\begin{cases} U_A(s) = R_A I_A(s) + L_A s I_A(s) + K\Phi\Omega(s) \\ Js\Omega(s) = K\Phi I_A(s) - f\Omega(s) - m_s \end{cases} \tag{4}$$

III. DC MOTOR DYNAMIC EQUIVALENT SCHEMATIC

Equation (4) can be rearranged so that the armature current and speed are evidenced:

$$\begin{cases} I_A(s) = [U_A(s) - K\Phi\Omega(s)]\dfrac{1}{R_A + L_A s} \\ \Omega(s) = [K\Phi I_A(s) - m_s]\dfrac{1}{Js + f} \end{cases} \tag{5}$$

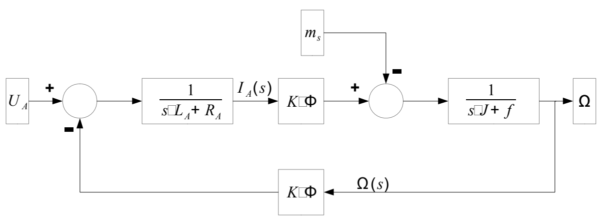

Figure 2. Block diagram of a DC motor

From equation (5) it can be developed the classical block diagram of a DC motor, indicated in figure 2.

Based on figure 2 and equation (4) and (5), the dynamic behaviour of the DC motor could be expressed by equation (6):

$$\begin{aligned} \Omega(s) &= \frac{K\Phi}{sJ+f} I_A(s) - \frac{m_s}{sJ+f} = \ldots \\ \ldots &= \frac{K\Phi}{sJ+f}\,\frac{U_A(s) - K\Phi\Omega(s)}{R_A + sL_A} - \frac{m_s}{sJ+f} = \ldots \\ \ldots &= \frac{K\Phi U_A(s) - (K\Phi)^2\Omega(s)}{(sJ+f)(R_A+sL_A)} - \frac{m_s}{sJ+f} \end{aligned} \tag{6}$$

When the disturbance torque is negligible compared with the voltage-speed behaviour and it can be neglected, equation (6) could be approximated by equation (7):

$$\Omega(s) = \frac{K\Phi U_A(s) - (K\Phi)^2\Omega(s)}{(sJ+f)(R_A+sL_A)} \tag{7}$$

Based on equation (7) it can be derived the transfer function of the DC motor in the hypothesis of neglecting the disturbance torque as indicated in equation (8):

$$\begin{aligned} \frac{\Omega(s)}{U_A(s)} &= \frac{K\Phi}{(sJ+f)(R_A+sL_A) + (K\Phi)^2} = \ldots \\ \ldots &= \frac{K\Phi}{s^2 L_A J + s(L_A f + R_A J) + R_A f + (K\Phi)^2} = \ldots \\ \ldots &= \frac{\dfrac{K\Phi}{L_A J}}{s^2 + s\dfrac{L_A f + R_A J}{L_A J} + \dfrac{R_A f + (K\Phi)^2}{L_A J}} = \\ &\frac{\dfrac{K\Phi}{L_A J}}{\sqrt{\dfrac{R_A f + (K\Phi)^2}{L_A J}}}\,\frac{\sqrt{\dfrac{R_A f + (K\Phi)^2}{L_A J}}}{s^2 + s\dfrac{L_A f + R_A J}{L_A J} + \dfrac{R_A f + (K\Phi)^2}{L_A J}} \end{aligned} \tag{8}$$

The result will be arranged in the form of a general 2nd system transfer function:

$$G(s) = K_1 \frac{\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{9}$$

By identifying terms from equations (8) and (9) the coefficients of the general transfer function are expressed as variables depending on the motor values:

$$\begin{aligned} K_1 &= \frac{\dfrac{K\Phi}{L_A J}}{\sqrt{\dfrac{R_A f + (K\Phi)^2}{L_A J}}}; \\ \omega_n &= \sqrt{\frac{R_A f + (K\Phi)^2}{L_A J}}; \\ 2\zeta\omega_n &= \frac{L_A f + R_A J}{L_A J} \end{aligned} \tag{10}$$

In equation (10) the terms R_A, L_A are the armature parameters that can be known from the nameplate of the motor. In equation (10) the parameters J and f, the dynamic mechanical parameters, are not known. Their expression is dependent on R_A, L_A and ω, ζ, as expressed in equation (11):

$$\begin{cases} J = \dfrac{R_A f + (K\Phi)^2}{L_A \omega_n^2} \\ f = \dfrac{(2\xi\omega_n L_A - R_A)(K\Phi)^2}{L_A^2\omega_n^2 + R_A^2 - 2\xi\omega_n L_A R_A} \end{cases} \quad \textbf{(11)}$$

IV. DC PARAMETER IDENTIFICATION

In the DC motor representation as a block schematic (figure 2) there are several parameters that have to be known for a dynamic simulation:

- Electric R_A, L_A
- Mechanic J, f
- Flux $K\Phi$

The electrical parameters can be determined by direct or indirect laboratory measurements. The flux constant $K\Phi$ is calculated from the nominal values of producer indicated parameters.

The proposed method relies on determining by measuring the maximum overshoot and the peak time.

For the experimental measurements it was used a DC motor with data indicated in Table 1.

Table 1. Nominal Data for the DC Motor

	Nominal data		
	Data	Value	Unit
1	U_A	200	V
2	I_A	2,0	A
3	n_n	1500	rot/min

The electrical data of the motor determined by direct measurement are indicated in Table 2.

Table 2. Electrical Data for the DC Motor

	Nominal data		
	Data	Value	Unit
1	R_A	15,8	Ω
2	L_A	0,41	H

The acquired transient response of the DC motor when applying the nominal voltage is indicated in figure 5.

From the acquired data it is determined the maximum transient value Ω_{max} and medium steady valued Ω_{med} of speed and the time of apparition of the maximum value t_p:

$$\begin{cases} t_p = 0.064\,\text{s} \\ \Omega_{\max} = 1600{,}50\, rad/s \\ \Omega_{med} = 1371{,}01\, rad/s \end{cases} \quad \textbf{(12)}$$

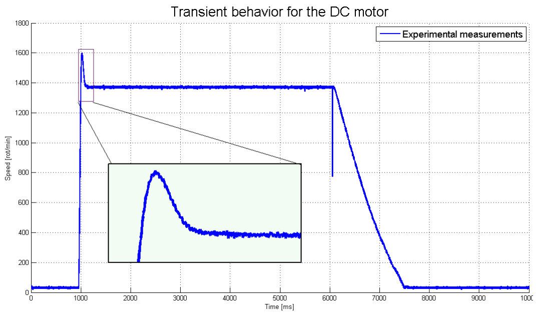

Figure. 5 The acquired transient response of the DC motor when applying the nominal voltage

The values indicated in equation (12) are compared and identified with the parameters of a generic 2nd order system when a step impulse is applied. The response equation *c(t)* when a step impulse is applied to a 2nd order system is given by equation (13):

$$\begin{cases} c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t - \dfrac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) \\ \omega_d = \omega_n\sqrt{1-\zeta^2} \end{cases} \quad \textbf{(13)}$$

From equation (13) it can be obtained the values of the overshoot and its time of apparition, by differentiation [11][12]:

$$\begin{cases} t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} \\ \sigma_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} \end{cases} \quad \textbf{(14)}$$

The values of equation (14) lead to the transient regime parameter values, expressed in equation (15):

$$\begin{cases} t_p = 0.0640\,\text{s} \\ \sigma_p = 0.1674 \end{cases} \quad \textbf{(15)}$$

From equations (15) and (12), the natural pulsation frequency and the dumping factor are expressed and determined as indicated in equation (16)[11]:

$$\begin{cases} \zeta = \sqrt{\dfrac{\left(\dfrac{\ln\sigma_p}{\pi}\right)^2}{1+\left(\dfrac{\ln\sigma_p}{\pi}\right)^2}} = 0{,}4945 \\ \omega_n = \dfrac{\pi}{t_p\sqrt{1-\zeta^2}} = 64{,}9776 \end{cases} \quad \textbf{(16)}$$

The dynamic parameters expressed in equation (15) are used to determine the values of parameters from equation (11).

$$\begin{cases} J = \dfrac{R_A f + (K\Phi)^2}{L_A \omega_n^2} = 0{,}0011 \\ f = \dfrac{(2\xi\omega_n L_A - R_A)(K\Phi)^2}{L_A^2\omega_n^2 + R_A^2 - 2\xi\omega_n L_A R_A} = 0{,}0279 \end{cases} \quad \textbf{(17)}$$

Equation (17) expresses the values of mechanical dynamical parameters of DC motor determined from transient behaviour when applied the nominal voltage.

Using the values of equation (17) in the block schematic of the motor (figure 2) it can be simulated the dynamic behaviour of the DC motor. Once the mechanical parameters are determined, there can be designed the rest of the elements of a control system: current, speed and position regulators.

V. EXPERIMENTAL VERIFICATION

For the experimental verification of this method, it was used a laboratory experimental test (figure 6), which consists of a DC motor, supply voltage and a data acquisition system for speed measurement in transient regime.

Figure 6. Experimental stand for data measuring and acquisition

The acquired data are analysed by a software program and the values of the mechanical parameters are calculated (equation 20). For experimental verification, in figure 7 it is indicated the graphical representations of the acquired data, the system behaviour represented as a block schematic using the measured data and the calculated behaviour based on the presented method.

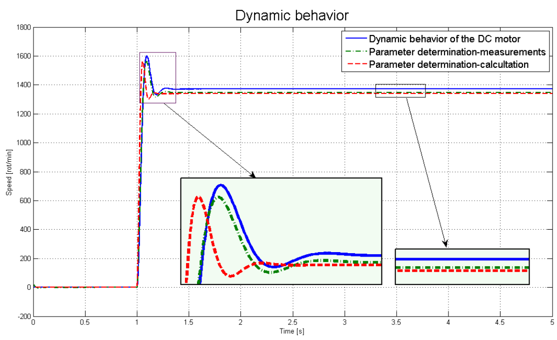

Figure 7. Comparison between dynamic behaviour of DC motor and dynamic representation based on parameter determination

The values in Table 3 correspond to measured and acquired values of speed when a step input signal is applied. Also Table 3 contains the speed overshoot and its time estimated by measurement and calculation based on the present method of determination. From the determined values, it is calculated several errors related to medium steady value and maximum overshoot for speed and its time of apparition.

Because of the low values of errors (below 5%) (Table 3) this method can be considered accurate enough for practical industrial applications. If more precision is required the presented method could be completed by means of reducing the error. Such methods of reducing the error could be based on artificial intelligence methods that have as starting points the calculated values obtained as indicated in the present paper, the calculated errors and the desired errors.

Table. 3. Determination errors

	Dynamic behaviour of DC motor	Dynamic representation based on parameter determination – measurement based	Dynamic representation based on parameter determination – calculation based
n [rot/min]	1371	1348	1339
n_{max} [rot/min]	1601	1562	1563
σ_p	0,1678	0,1587	0,1673
t_p [s]	1,094	1,090	1,056
ε_n	-	-1,71%	-2,39%
$\varepsilon_{n_{max}}$	-	-2,50%	-2,43%
ε_{σ_p}	-	-5,73%	-0,30%
ε_{t_p}	-	-0,37%	-3,60%

VI. CONCLUSION

In this paper it was presented a different method of determining the mechanical dynamic parameters of a DC motor starting from processing the dynamic behaviour data when nominal voltage is applied. Speed dynamical data was acquired from the starting procedure of the DC motor. The behaviour of the dynamic regime of the start of DC motor is measured and based on the presented algorithm the dynamic mechanical parameters were calculated. Parameter calculation based on the presented method leads to a dynamic representation with low errors (< 5%) as far as speed overshooting, peak time and permanent regime speed. In order to use this method, it is not necessary a special testing stand. The method only requires the analysed DC motor, a data acquisition system and processing software.

The calculated parameters can be used with good precision in estimating transient and permanent regime behaviour of the DC motor. Also dynamic mechanical parameters calculated this way are important for designing DC motor driving systems in open or closed-loop configurations.

In order to apply the presented method for mechanical dynamic parameters it is needed only a tachogenerator and an acquisition system. The calculation of parameters could be done with any program that could implement basic mathematical operations (it could work with Math OpenOffice® module, or Scilab® package) Thus this method is a very cheap one and could be implemented without any difficulties in any measuring laboratory.

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